Despite that disappointing result, it must be noticed that below a lift coefficient of 0.40, a lower drag (10%) is observed for the crescent wing (Λ_{ma} = 22.3 deg) in comparison with the basic elliptic wing. This effect is presumed to arise from the spanwise deflection of the surface streamlines over the crescent wing, with a possible reduction of the chordwise component of the friction drag.

Another positive result is the higher maximum lift coefficient resulting from the outboard leading-edge sweep angle. Whether the two mentioned benefits of the crescent wing explain its selection by several species is an open question. Many other properties of animal lifting surfaces should be taken into account, such as flexibility, before attempting to answer the question.

This work was initiated with the hope of a possible lift-induced pressure drag reduction. From this point of view it appears that only very small (second-order) effects, positive or negative, can be expected from deviations with respect to the basic Prandtl's definition of minimum induced drag, and we can ask if the theoretical splitting between viscous and inviscid drag has to be maintained. Two reasons may be put forward. First, the assumptions required to split the drag components from wind-tunnel data involve a level of uncertainty of the same order as, or even greater than, the expected changes of the induced drag. Second, what the aeronautical engineer finally needs is a lower total drag, irrespective of the balance between the components.

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Wake Curvature and Airfoil Lift

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Introduction

RECENTLY, one witnesses renewed interest in the effects of wake curvature upon the flow around airfoils; sometimes, however, puzzling results are reported.[1] The purpose of this Note will be to discuss, in the light of some previous results,[2] the physics of the phenomena, which are frequently obscured by emphasis on numerics. An analogy with the jet-flap will be put forward in this context, which will throw more light upon the subject and allow assessing the relative importance of the effects involved.

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Effects of Boundary Layer and Wake upon the Lift Slope

This is a problem as old as aerodynamics itself: it is known that the lift slope of an airfoil, in incompressible flow, at moderate angles of attack (i.e., before separation sets in), is fairly well predicted by potential theory (without considering viscous effects), with only a small correction factor:

$$C_{L\alpha} = (1 - \eta)C_{L\alpha(\mathrm{pot})} \tag{1}$$

where, for most airfoils, $C_{L\alpha(\mathrm{pot})}$ is very close to 2π, and η is of the order of 0.1, depending slightly on the Reynolds number. Certainly, this loss is due to the development of the boundary layer, but the explicit mechanism is not obvious; sometimes, the effect of wake curvature is invoked. Therefore, the aim of the following will be to render things more explicit.

Currently, analysis of the real flow around an airfoil follows two approaches: 1) either a viscous-inviscid coupling of a potential (or Euler) solution to a boundary-layer computation (with possible iterations); or 2) a full frontal attack with the Navier-Stokes equations. While potentially able to furnish the ultimate answer, in our opinion this latter approach suffers two major drawbacks. On the one hand, even with modern hypercomputers, it is still not possible to refine the computational grids to the point of getting rid of artificial viscosity terms (which, in our belief, render questionable the whole endeavor); and, besides, this "pointillist" way of painting the reality often renders hazy the qualitative behavior of the flow and hinders attempts at reasoned modifications of the airfoil shape, to produce certain desired performance features. Subsequently, both approaches always furnish values of $C_{L\alpha}$ quite close to experiment,[3] but do not provide an explanation of the facts. One point which has frequently been raised in this connection is the influence of the wake curvature[1,4]; before discussing this question some basic theorems will be recalled.

Wake-Curvature Singularity in Potential Flow

On a lift-carrying airfoil, the streamline emerging from the trailing edge always exhibits a certain curvature, $1/\bar{r}_s$. In Ref. 2, a formula has been deduced for this quantity, depending on the flow potential $F(z)$, and on the complex function $z = f(\zeta)$, which maps the airfoil contour onto the unit circle. It has been further proved that, at the trailing edge itself, the streamline curvature is infinite (in potential flow) unless the airfoil is at a certain, singular, angle of attack α^*, whose value is proper to each particular airfoil. Furthermore, and adopting Lighthill's approach, if one considers the equivalent blunt airfoil contour, obtained by adding the boundary-layer displacement thickness to the basic section (Fig. 1) one can, in principle, write down its conformal mapping function (for a discussion of the conformal mapping of blunt—or open—contours, see Ref. 2); then the same formulas apply, and one finds the curvature along the trailing-edge streamlines. Now, based on physical reasoning, Proposition IX of Ref. 2 states that, at each angle of attack α, the interplay between the

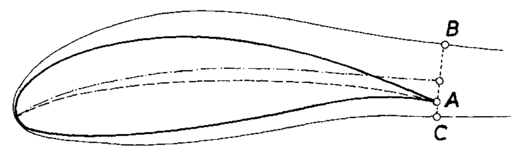

Fig. 1 Equivalent airfoil contour, generated by adding the boundary-layer thickness to the original shape.

outer potential flow and the boundary-layer modifies the pressure distribution and the boundary-layer thickness, to finally impart such a shape to the equivalent contour as to render the streamline curvature at the trailing edge finite: i.e., the singular α^* of the resulting contour always coincides with the actual angle of attack.

Analogy with the Jet-Flap

If a streamline is curved, a normal pressure gradient develops; but, to avoid possible confusion, it must be stressed that such a gradient is present everywhere within the flowfield, even in potential flow, and there is nothing to worry about. No supplementary terms have to be introduced in order to account for the effects of streamline curvature. However, within the boundary layer and the wake, the situation changes, as the Bernoulli constant is not the same throughout the flowfield. One way to understand the phenomena involved is to introduce an analogy with a jet-flap: i.e., the wake emerging from the trailing edge may be imagined to act as a jet having a negative momentum, equal precisely to the airfoil drag, $J = -\rho/2U_\infty^2C_D$. We note that this analogy has been first put to light as early as 1957.[5]

Then, any theory of jet-flapped airfoils may be used to assess the effect upon the lift of the wake curvature: since the equivalent jet intensity is weak, a simplified method would do the job. Such a theory was developed in Ref. 6; there, the jet was replaced by a vortex sheet, along which a circulation distribution is placed, related to the pressure differential Δp across the jet. This normal force is supported by the jet through the deflection of its momentum vector $\boldsymbol{J}$, and here the local streamline (or wake) curvature enters into play:

$$\Delta p = J/\bar{r}_s \tag{2}$$

As, in our case, the equivalent jet coefficient is negative $(-C_D)$, the vortices distributed along the wake have negative intensities; finally, one finds for the lift slope:

$$C_{L\alpha} \cong 2\pi\alpha(1 - C_D/4) \tag{3}$$

In the absence of massive separation, for typical airfoils, C_D is of the order of 0.01, therefore, the above negative correction (due only to the wake curvature) is quite small. We remark that, from the data of Ref. 1 it would appear that taking the wake curvature into account would produce an increase in lift, a result which is difficult to understand.

Discussion

Obviously, the effect upon the lift of the vorticity distribution along the wake, due to its curvature as predicted by the jet-flap analogy, is small and can be neglected, for all intents and purposes. Then, two questions arise:

The first question is what is the reason for this loss of lift? In fact, almost all procedures embodying viscous-inviscid coupling provide fair estimates of the lift slope,[3] even without taking the wake-curvature effects explicitly into account. A physical image of the phenomena involved may be envisaged as follows: due to the pressure distribution, the boundary-layer development is such that, usually, at the trailing edge (where local separation is often present), its upper surface thickness is much larger than on the lower surface (Fig. 1) and, in addition, this dissymmetry increases with α. Considering now the skeleton of the equivalent (blunt) airfoil, one may see that it has a lower incidence and less camber than the original contour, thus producing less lift. The requirement for removal of wake-curvature singularity (see above) results in the equivalent contour having, in addition, less aft-loading[4] and a negative effective camber towards the trailing edge,[2] which also contribute to the lift loss. We conclude, therefore, that the reduction in lift slope can be accounted for by the effect of the boundary-layer thickness alone, without invoking the curvature of the wake.

The second question to be addressed is what is then, albeit small, the main effect of the wake curvature to be embodied in a consistent theory? The answer lies in the way in which the Kutta condition is to be imposed. At the physical trailing edge (point A in Fig. 1), the pressure must be single-valued, p_A; then, one of the basic assumptions of boundary-layer theory (constant pressure across the thickness), would require that the same pressure should prevail also at points B and C, the upper- and lower-surface edges of the equivalent blunt contour. It is here that the wake curvature enters into play, for a correct formulation of the Kutta condition: namely, between the edges of the wake, a pressure differential, $p_B - p_C$, has to be imposed:

$$p_B - p_C = \rho U_\infty^2 C_D/(2\bar{r}_s) \tag{4}$$

This implies, of course, departing from the first-order boundary-layer theory which, at the trailing edge, becomes questionable anyway. How to implement the ideas put forward here is, however, another story.

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Tail Load Calculations for Light Airplanes

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Introduction

MOST airplanes have a download on their rear tail in order to balance the nose-down pitching moment produced by their cambered wing. This is the usual case, because the wing profile curvature required for good wing stall unfortunately produces a nose-down pitching moment. The negative pitching moment coefficient $(C_M < 0)$ is a pure couple that is independent of the unstalled lift coefficient $(C_L < C_{L,\max})$, and remains constant for any center of gravity (c.g.) location. As shown in Fig. 1, this nose-down couple must be

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